Integration of Choice and Latent Variable Models

by

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ABSTRACT
This paper presents a general methodology and framework for including latent variables—in particular, attitudes and perceptions—in choice models. This is something that has long been deemed necessary by behavioral researchers, but is often either ignored in statistical models, introduced in less than optimal ways (e.g., sequential estimation of a latent variable model then a choice model, which produces inconsistent estimates), or introduced for a narrowly defined model structure. The paper is focused on the use of psychometric data to explicitly model attitudes and perceptions and their influences on choices. The methodology requires the estimation of an integrated multi-equation model consisting of a discrete choice model and the latent variable model's structural and measurement equations. The integrated model is estimated simultaneously using a maximum likelihood estimator, in which the likelihood function includes complex multi-dimensional integrals. The methodology is applicable to any situation in which one is modeling choice behavior (with any type and combination of choice data) where (1) there are important latent variables that are hypothesized to influence the choice and (2) there exist indicators (e.g., responses to survey questions) for the latent variables. Three applications of the methodology provide examples and demonstrate the flexibility of the approach, the resulting gain in explanatory power, and the improved specification of discrete choice models.

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Introduction

Recent work in discrete choice models has emphasized the importance of the explicit treatment of psychological factors affecting decision-making. (See, for example, Koppelman and Hauser, 1979; McFadden, 1986a; Ben-Akiva and Boccara, 1987; Ben-Akiva, 1992; Ben-Akiva et al., 1994; Morikawa et al., 1996.) A guiding philosophy in these developments is that the incorporation of psychological factors leads to a more behaviorally realistic representation of the choice process, and consequently, better explanatory power.

This paper presents conceptual and methodological frameworks for the incorporation of latent factors as explanatory variables in choice models. The method described provides for explicit treatment of the psychological factors affecting the decision-making process by modeling them as latent variables. Psychometric data, such as responses to attitudinal and perceptual survey questions, are used as indicators of the latent psychological factors. The resulting approach integrates choice models with latent variable models, in which the system of equations is estimated simultaneously. The simultaneous estimation of the model structure represents an improvement over sequential methods, because it produces consistent and efficient estimates of the parameters. (See Everitt, 1984 and Bollen, 1989 for an introduction to latent variable models and Ben-Akiva and Lerman, 1985 for a textbook on discrete choice models.)

Three applications of the methodology are presented to provide conceptual examples as well as sample equations and estimation results. The applications illustrate how psychometric data can be used in choice models to improve the definition of attributes and to better capture taste heterogeneity. They also demonstrate the flexibility and practicality of the methodology, as well as the potential gain in explanatory power and improved specifications of discrete choice models.

Supporting Research

Discrete choice models have traditionally presented an individual's choice process as a black box, in which the inputs are the attributes of available alternatives and individual characteristics, and the output is the observed choice. The resulting models directly link the observed inputs to the observed output, thereby assuming that the inner workings of the black box are *implicitly* captured by the model. For example, discrete choice models derived from the random utility theory do not model explicitly the formation of attitudes and perceptions. The framework for the random utility choice model is shown in Figure 1. This figure, as well as the remaining figures in the paper, follows the convention of depicting a path diagram where the terms in ellipses represent *unobservable* (i.e. latent) constructs, while those in rectangles represent *observable* variables. Solid arrows represent *structural equations* (cause-and-effect relationships) and dashed arrows represent *measurement equations* (relationships between observable indicators and the underlying latent variables).

There has been much debate in the behavioral science and economics communities on the validity of the assumptions of utility theory. Behavioral researchers have stressed the importance of the cognitive workings inside the black box on choice behavior (see, for example, Abelson and Levy, 1985 and Olson and Zanna, 1993), and a great deal of research has been conducted to uncover cognitive anomalies that appear to violate the basic axioms of utility theory (see, for example, Gärling, 1998 and Rabin, 1998). McFadden (1997) summarizes these anomalies and argues that "most cognitive anomalies operate through errors in perception that arise from the way information is stored, retrieved, and processed" and that "empirical study of economic behavior would benefit from closer attention to how perceptions are formed and how they influence decision-making." To address such issues, researchers have worked to enrich choice models by modeling the cognitive workings inside the black box, including the explicit incorporation of factors such as attitudes and perceptions.

A general approach to synthesizing models with latent variables and psychometric-type measurement models has been advanced by a number of researchers including Keesling (1972), Joreskog (1973), Wiley (1973), and Bentler (1980), who developed the structural and measurement equation framework and methodology for specifying and estimating latent variable models. Such models are widely used to measure unobservable factors. Estimation is performed by minimizing the discrepancy between (a) the covariance matrix of observed variables and (b) the theoretical covariance matrix predicted by the model structure, which is a function of the unknown parameters. Much of this work focuses on continuous latent constructs and continuous indicators. When discrete indicators are involved, direct application of the approach used for continuous indicators results in inconsistent estimates. For the case of discrete indicators, various corrective procedures can be applied. Olsson (1979), Muthen (1979, 1983, and 1984), and others developed procedures based on the application of polychoric correlations (rather than the Pearson correlations used for continuous indicators) to estimate the covariance matrix of the latent continuous indicators from the discrete indicators. Consistent estimates of the parameters can then be obtained by minimizing the discrepancy between this estimated covariance matrix and the theoretical covariance matrix. (See Bollen, 1989, for more discussion of discrete indicators.) Estimation methods for the case of discrete latent variables and discrete indicators was developed by Goodman (1974)—see McCutcheon (1987) for a discussion.

In the area of choice modeling, researchers have used various techniques in an effort to explicitly capture psychological factors in choice models. One approach applied is to include *indicators* of psychological factors (such as responses to survey questions regarding individuals' attitudes or perceptions) directly in the utility function as depicted in Figure 2 (see, for example, Koppelman and Hauser, 1979; Green, 1984; Harris and Keane, 1998).

Another frequently used approach is to first perform factor analysis on the indicators, and then use the fitted latent variables in the utility, as shown in Figure 3. (See, for example, Prashker, 1979a,b; and Madanat et al., 1995). Note that these fitted variables contain

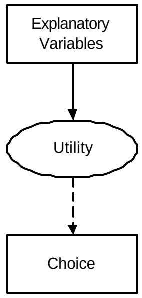

Figure 1:
Random Utility Choice Model

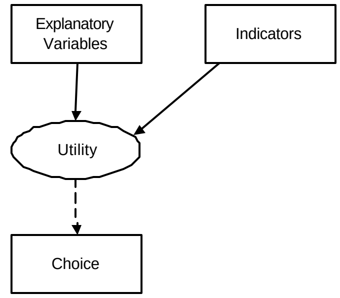

Figure 2:
Choice Model with Indicators Directly Included in Utility

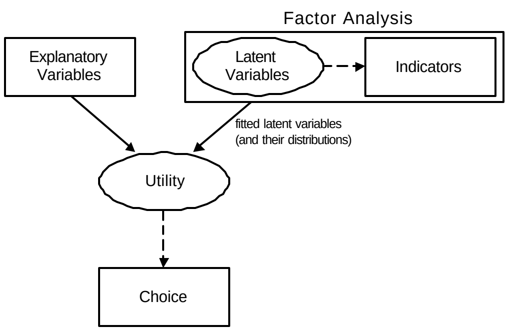

Figure 3:
Sequential Estimation: Factor Analysis followed by a Choice Model

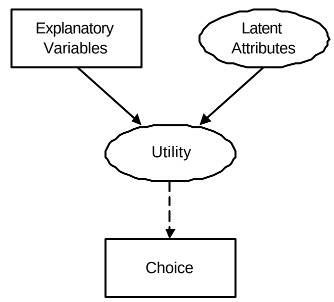

Figure 4:
Choice Model with Latent Attributes

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measurement error, and so to obtain consistent estimates, the choice probability must be integrated over the distribution of the latent variables, where the distribution of the factors is obtained from the factor analysis model. (See, for example, Morikawa, 1989.)

Other approaches have been developed in market research (in an area called *internal market analysis*), in which both latent attributes of the alternatives and consumer preferences are inferred from preference or choice data. (For a review of such methods, see Elrod, 1991; and Elrod and Keane, 1995.) For example, Elrod 1988 and 1998, Elrod and Keane 1995, and Keane 1997 develop random utility choice models (multinomial logit and probit) that contain latent attributes. In estimating these models, they do not use any indicators other than the observed choices. Therefore, the latent attributes are alternative specific and do not vary among individuals in a market segment. However they do use perceptual indicators post-estimation to aid in interpretation of the latent variables. The framework for their model is shown in Figure 4.

Wedel and DeSarbo (1996) and Sinha and DeSarbo (1997) describe a related method based on multidimensional scaling.

This research extends the above-described methods by formulating a general treatment of the inclusion of latent variables in discrete choice models. The formulation incorporates psychometric data as indicators of the latent variables. We employ a simultaneous maximum likelihood estimation method for integrated latent variable and discrete choice models, which results in consistent and efficient estimates of the model parameters. The formulation of the integrated model and the simultaneous estimator are described in the following sections of the paper.

Our work on this methodology began during the mid-1980s with the objective of making the connection between econometric choice models and the extensive market research literature on the study of consumer preferences (Cambridge Systematics, 1986; McFadden, 1986a; and Ben-Akiva and Boccara, 1987). We first developed a unifying framework for the incorporation of subjective psychometric data in individual choice models. We then proceeded to undertake a number of empirical case studies, some of which are described in this paper. Finally, in this paper, we present a general specification and estimation method for the integrated model.

The Methodology

The objective of this research is the integration of latent variable models, which aim to operationalize and quantify unobservable concepts, with discrete choice models. The integrated model is employed to include latent variables in choice models. The methodology incorporates indicators of the latent variables provided by responses to survey questions to aid in estimating the model. A simultaneous estimator is used, which results in latent variables that provide the best fit to both the choice and the latent variables indicators.

Notation

The following notation, corresponding to choice model notation, is used:

X observed variables, including
- S characteristics of the individual
- Z_i attributes of alternative *i*

X^* latent (unobservable) variables, including
- S^* latent characteristics of the individual
- Z_i^* latent attributes of alternative *i* as perceived by the individual

I indicators of X^* (e.g., responses to survey questions related to attitudes, perceptions, etc.)
- I_S indicators of S^*
- I_{Z_i} indicators of Z_i^*

U_i utility of alternative *i*

U	vector of utilities
y_i	choice indicator; equal to 1 if alternative i is chosen and 0 otherwise
y	vector of choice indicators
a, b, g	unknown parameters
$\eta, \varepsilon, \upsilon$	random disturbance terms
Σ, σ	covariances of random disturbance terms
D	generic distribution

Framework and Definitions

The integrated modeling framework, shown in Figure 5, consists of two components, a choice model and a latent variable model.

As with any random utility choice model, the individual's utility U for each alternative is assumed to be a latent variable, and the observable choices y are *manifestations* of the underlying utility. Such observable variables that are manifestations of latent constructs are called *indicators*. A dashed arrow representing a *measurement equation* links the unobservable U to its observable indicator y. Solid arrows representing *structural equations* (i.e., the cause-and-effect relationships that govern the decision making process) link the observable and latent variables (X, X^*) to the utility U.

It is possible to identify a choice model with limited latent variables using only observed choices and no additional indicators (see, e.g., Elrod, 1998). However, it is quite likely that the information content from the choice indicators will not be sufficient to empirically identify the effects of individual-specific latent variables. Therefore, indicators of the latent variables are used for identification, and are introduced in the form of a latent variable model.

The top portion of Figure 5 is a latent variable model. Latent variable models are used when we have available indicators for the latent variables X^*. Indicators could be responses to survey questions regarding, for example, the level of satisfaction with or importance of attributes. The figure depicts such indicators I as manifestations of the underlying latent variable X^*, and the associated measurement equation is represented by a dashed arrow. A structural relationship links the observable causal variables X (and potentially other latent causal variables X^*) to the latent variable X^*.

The integrated choice and latent variable model explicitly models the latent variables that influence the choice process. Structural equations relating the observable explanatory variables X to the latent variables X^* model the behavioral process by which the latent variables are formed. While the latent constructs are not observable, their effects on indicators are observable. The indicators allow identification of the latent constructs. They also contain information and thus potentially provide for increased efficiency in model estimation. Note that the indicators do not have a causal relationship that influences the behavior. That is, the arrow goes *from* the latent variable *to* the indicator, and the indicators are only used to aid in measuring the underlying causal relationships (the solid arrows). Because the indicators are not part of the causal relationships, they are typically used only in the model estimation stage and not in model application.

General Specification of the Model

As described above, the integrated model is composed of two parts: a discrete choice model and a latent variable model. Each part consists of one or more structural equations and one or more measurement equations. Specification of these equations and the likelihood function follow.

Structural Equations

For the latent variable model, we need the distribution of the latent variables given the observed variables, $f_1(X^* | X; \gamma, \Sigma_\eta)$. For example:

$$X^* = h(X;g) + h \quad \text{and} \quad \eta \sim D(0, \Sigma_\eta) \tag{1}$$

This results in one equation for each latent variable.

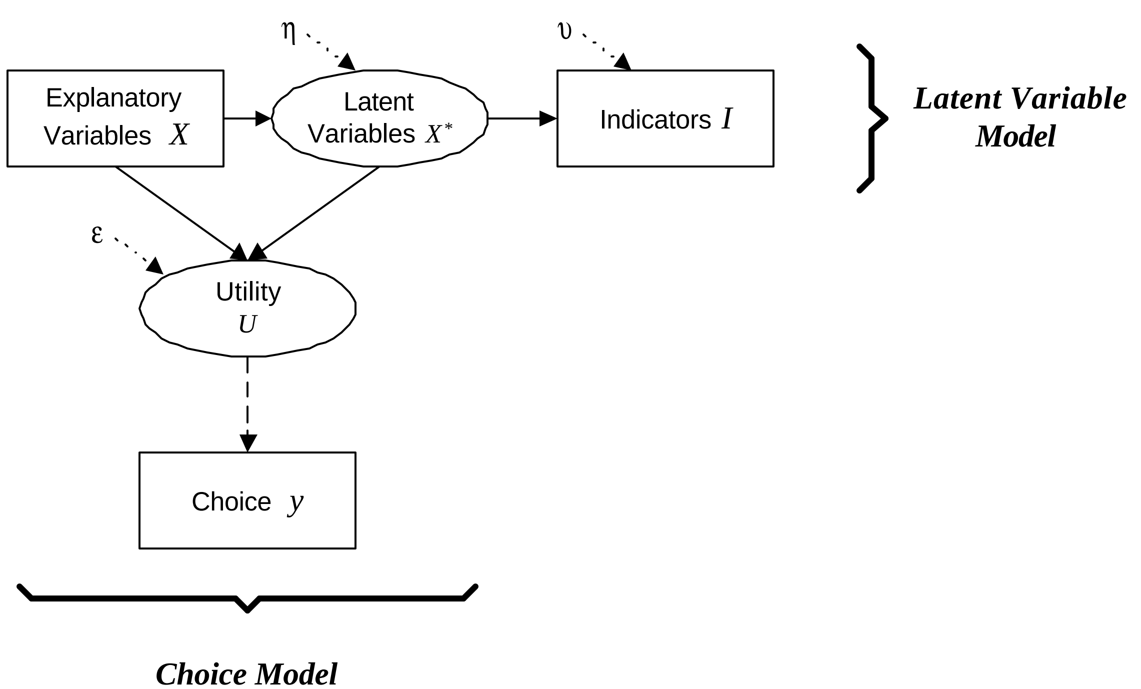

Figure 5: Integrated Choice and Latent Variable Model

For the choice model, we need the distribution of the utilities, $f_2(U|X, X^*; \beta, \Sigma_\varepsilon)$. For example:

$$U = V(X, X^*; b) + e \quad \text{and} \quad \varepsilon \sim D(0, \Sigma_\varepsilon) \tag{2}$$

Note that the random utility is decomposed into systematic utility and a random disturbance, and the systematic utility is a function of both observable and latent variables.

Measurement Equations

For the latent variable model, we need the distribution of the indicators conditional on the values of the latent variables, $f_3(I|X, X^*; \alpha, \Sigma_\upsilon)$. For example:

$$I = g(X, X^*; a) + u \quad \text{and} \quad \upsilon \sim D(0, \Sigma_\upsilon) \tag{3}$$

This results in one equation for each indicator (i.e. each survey question). These measurement equations usually contain only the latent variables on the right-hand-side. However, they may also contain individual characteristics or any other variable determined within the model system such as the choice indicator. In principle, such parameterizations can be allowed to capture systematic response biases when the individual is providing indicators. For example, in a brand choice model with latent product quality (Z^*), one may include the indicator y_i for the chosen brand, for example, $I_r = \alpha_{1r} Z_i^* + \alpha_{2r} y_i + \upsilon_r$, where I_r is an indicator of the perceived quality of alternative i. This would capture any exaggerated responses in reporting the perceived quality of the chosen brand, perhaps caused by justification bias.

For the choice model, we need to express the choice as a function of the utilities. For example, assuming utility maximization:

$$y_i = \begin{cases} 1, \text{ if } U_i = \max_j \{U_j\} \\ 0, \text{ otherwise} \end{cases} \tag{4}$$

Note that $h(\cdot)$, $V(\cdot)$, and $g(\cdot)$ are functions, which are currently undefined. Typically, as in our case studies, the functions are specified to be linear in the parameters, but this is not necessary. Also note that the distribution of the error terms must be specified, leading to additional unknown parameters (the covariances, S). The covariances often include numerous restrictions and normalizations to both simplify the model and provide identification.

Integrated Model

The integrated model consists of equations (1) to (4). Equations (1) and (3) comprise the latent variable model, and equations (2) and (4) comprise the choice model. From equations (2) and (4) and an assumption about the distribution of the disturbance, e, denoted as

$f_2(U|X, X^*; \beta, \Sigma_\varepsilon)$, we derive $P(y|X, X^*; \beta, \Sigma_\varepsilon)$, the choice probability conditional on both observable and latent explanatory variables.

Likelihood Function

We use maximum likelihood techniques to estimate the unknown parameters. The most intuitive way to create the likelihood function for the integrated model is to start with the likelihood of a choice model without latent variables:

$$P(y|X; \beta, \Sigma_\varepsilon) \tag{5}$$

The choice model can be any number of forms, e.g., logit, nested logit, probit, ordered probit, and can include the combination of different choice indicators such as stated and revealed preferences.

Now we add the latent variables to the choice model. Once we hypothesize an unknown latent construct, X^*, its associated distribution, and independent error components (η, ε), the likelihood function is then the integral of the choice model over the distribution of the latent constructs:

$$P(y|X; \beta, \gamma, \Sigma_\varepsilon, \Sigma_\eta) = \int_{X^*} P(y|X, X^*; \beta, \Sigma_\varepsilon) f_1(X^*|X; \gamma, \Sigma_\eta) dX^* \tag{6}$$

We introduce indicators to improve the accuracy of estimates of the structural parameters. Assuming the error components $(\eta, \varepsilon, \upsilon)$ are independent, the joint probability of the observable variables *y* and *I*, conditional on the exogenous variables *X*, is:

$$\begin{aligned} &f_4(y, I|X; \alpha, \beta, \gamma, \Sigma_\varepsilon, \Sigma_\upsilon, \Sigma_\eta) = \\ &\qquad \int_{X^*} P(y|X, X^*; \beta, \Sigma_\varepsilon) f_3(I|X, X^*; \alpha, \Sigma_\upsilon) f_1(X^*|X; \gamma, \Sigma_\eta) dX^* \end{aligned} \tag{7}$$

Note that the first term of the integrand corresponds to the choice model, the second term corresponds to the measurement equation from the latent variable model, and the third term corresponds to the structural equation from the latent variable model. The latent variable is only known to its distribution, and so the joint probability of y, *I*, and X^* is integrated over the vector of latent constructs X^*.

Functional Forms

The forms of the variables (e.g. discrete or continuous) and assumptions about the disturbances of the measurement and structural equations determine the functional forms in the likelihood equation. Frequently we assume linear in the parameter functional forms, and disturbances that have normal (or extreme value for the choice model) distributions.

The choice model portion of the likelihood function is a standard choice model, except that the utility is a function of latent constructs. The form of the probability function is derived from equations (2) and (4) and an assumption about the distribution of the disturbance, ε. For example, for a choice of alternative *i*:

$$U_i = V_i + e_i \text{ and } V_i = V_i(X, X^*; b) \ , \ \ i \in C \text{, } C \text{ is the choice set}$$

$$\begin{aligned} P(y_i = 1 | X, X^*; \beta, \Sigma_\varepsilon) &= P(U_i \geq U_j, \forall j \in C) \\ &= P(V_i + e_i \geq V_j + e_j, \forall j \in C) \\ &= P(\varepsilon_j - \varepsilon_i \leq V_i - V_j, \forall j \in C) \end{aligned}$$

If the disturbances, e, are iid standard Gumbel, then:

$$P(y_i = 1 | X, X^*; \beta) = \frac{e^{V_i}}{\sum_{j \in C} e^{V_j}} \qquad \text{[Logit Model]}$$

Or, in a binary choice situation with normally distributed disturbances:

$$P(y_i = 1 | X, X^*; \beta) = \Phi(V_i - V_j) \qquad \text{[Binary Probit Model]}$$

where Φ is the standard normal cumulative distribution function

The choice model can take on other forms. For example, ordered categorical choice indicators would result in either ordered probit or ordered logistic form (e.g., see Case Study 3 of this paper).

The form of the distribution of the latent variables is derived from equation (1); the form of the distribution of the indicators is derived from equation (3). The disturbances of the structural and measurement equations of the latent variable model are often assumed to be normally and independently distributed. The latent variables are assumed to be orthogonal and the indicators are assumed to be conditionally (on X^* and *X*) independent. In this case, the resulting densities are:

$$f_1(X^* | X; \gamma, \sigma_\eta) = \prod_{l=1}^{L} \frac{1}{\sigma_{\eta_l}} \phi\left(\frac{X_l^* \text{-} h(X;\gamma_l)}{\sigma_{\eta_l}} \right)$$

$$f_3(I | X, X^*; \alpha, \sigma_\upsilon) = \prod_{r=1}^{R} \frac{1}{\sigma_{\upsilon_r}} \phi\left(\frac{I_r \text{-} g(X,X^*;\alpha_r)}{\sigma_{\upsilon_r}} \right)$$

where:

ϕ is the standard normal density function

s_{u_r} and s_{h_l} are the standard deviations of the error terms of u_r and h_l, respectively

R is the number of indicators

L is the number of latent variables

Both the indicators and the latent variables may be either discrete or continuous. See Gopinath (1995) and Ben-Akiva and Boccara (1995) for details on the specification and estimation of models with various combinations of discrete and continuous indicators and latent constructs.

Theoretical Analysis

The methodology presented here improves upon the techniques described by Figures 1 through 4.

Figure 1 - Omitting important latent variables may lead to mis-specification and inconsistent estimates of all parameters.

Figure 2 - We a priori reject the use of the indicators directly in the choice model – they are not causal, they are highly dependent on the phrasing of the survey question, and, furthermore, they are not available for forecasting.

Figure 3a - The two-stage sequential approach without integration leads to measurement errors and results in inconsistent estimates.

Figure 3b - The two-stage sequential approach with integration results in consistent, but inefficient estimates. As long as one is integrating (and therefore, by necessity, not using a canned estimation procedure) one may as well estimate the model simultaneously.

Figure 4 - The choice and latent variable model without indicators is restrictive in that the latent variables are alternative specific and cannot vary among individuals.

In summary, the approach we present is theoretically superior: it is a generalization of Figures 1 and 4 (so cannot be inferior) and it is statistically superior to sequential methods 3a and 3b. How much better is the methodology in a practical sense? The answer will vary based on the model and application at hand: in some cases it will not make a difference and, presumably, there are cases in which the difference will be substantial.

Identification

As with all latent variable models, identification is certainly an issue in these integrated choice and latent variable models. While identification has been thoroughly examined for special cases of the integrated framework presented here (see, e.g, Elrod 1988 and Keane 1997), necessary and sufficient conditions for the general integrated model have not been developed. Identification of the integrated models needs to be analyzed on a case-by-case basis.

In general, all of the identification rules that apply to a traditional latent variable model are applicable to the latent variable model portion of the integrated model. See Bollen (1989) for a detailed discussion of these rules. Similarly, the normalizations and restrictions that apply to a standard choice model would also apply here. See Ben-Akiva and Lerman (1985) for further information.

For the integrated model, a sufficient, but not necessary, condition for identification can be obtained by extending the *Two-step Rule* used for latent variable models to a *Three-step Rule* for the integrated model:

1. Confirm that the measurement equations for the latent variable model are identified (using, for example, standard identification rules for factor analysis models).

2. Confirm that, given the latent variables, the structural equations of the latent variable model are identified (using, for example, standard rules for a system of simultaneous equations).

3. Confirm that, given the distribution of the latent variables, the choice model is identified (using, for example, standard rules for a discrete choice model).

An ad-hoc method for checking identification is to conduct Monte Carlo experiments by generating synthetic data from the specified model structure (with given parameter values), and then attempt to reproduce the parameters using the maximum likelihood estimator. If the parameters cannot be reproduced to some degree of accuracy, then this is an indication that the model is not identified (or there is a coding error).

Another useful heuristic is to use the Hessian of the log-likelihood function to check for local identification. If the model is locally identified at a particular point, then the Hessian will be positive definite at this point. The inverse Hessian is usually computed at the solution point of the maximum likelihood estimator to generate estimates of the standard errors of estimated parameters, and so in this case the test is performed automatically.

Estimation

Maximum likelihood techniques are used to estimate the unknown parameters of the integrated model. The model estimation process maximizes the logarithm of the sample likelihood function over the unknown parameters:

$$\max_{\alpha,\beta,\gamma,\Sigma} \sum_{n=1}^{N} \ln f_4\left(y_n, I_n | X_n; \alpha, \beta, \gamma, \Sigma\right) \tag{8}$$

The likelihood function includes complex multi-dimensional integrals, with dimensionality equal to that of the integral of the underlying choice model plus the number of latent variables. There are three basic ways of estimating the model: a sequential numerical approach, a simultaneous numerical approach, and a simulation approach.

The sequential estimation method involves first estimating the latent variable model (equations 1 and 3) using standard latent variable estimators. The second step is to use fitted latent variables *and their distributions* to estimate the choice model, in which the choice probability is integrated over the distribution of the latent variables. The two step estimation method results in consistent, but inefficient estimates. See McFadden (1986a), Train et al. (1986), and Morikawa et al. (1996) for more details on the sequential approach.

An important point is that a sequential estimation procedure that treats the fitted latent variables as non-stochastic variables in the utility function introduces measurement error and results in inconsistent estimates of the parameters. If the variance of the latent variable's random error (η) is small, then increasing the sample size may sufficiently reduce the measurement error and result in acceptable parameter estimates. Increasing the sample size results in a more precise estimate of the expected value of the latent variable, and a small variance means that an individual's true value of the latent variable will not be too far off from the expected value. Train et al. (1986) found that for a particular model (choice of electricity rate schedule) the impact of the inconsistency on parameter estimates was negligible in a 3000 observation sample. However, this result cannot be generalized; the required size of the dataset is highly dependent on the model specification, and it requires that the variance of the latent variable's error (η) be sufficiently small. Note that the sample size has no effect on the variance of η. In other words, the measurement errors in the fitted latent variables do not vanish as the sample size becomes very large. Therefore, without running tests on the degree of inconsistency, it is a questionable practice to estimate these integrated choice and latent variable models by chaining a canned latent variable model software package with a canned choice model package. Performing these tests requires integration of the choice model. The first case study presented in this paper uses the sequential estimation approach. This case involved a small choice set, and it was necessary to integrate the choice probability over the latent variables.

The inconsistency issue already makes application of the sequential estimation approach quite complex, and it produces inefficient estimates. Alternatively, a fully efficient estimator can be obtained by jointly estimating equations (1) through (4). This involves programming the joint likelihood function (equation 8) directly in a flexible estimation package, which, ideally, has built in numerical integration procedures. This is the method that is used in the second and third case studies presented in this paper. The dimensionalities of the likelihoods are such that numerical integration is feasible and preferred.

As the number of latent variables increases, numerical integration methods quickly become infeasible and simulation methods must be employed. Typical estimation approaches used are Method of Simulated Moments or Simulated Maximum Likelihood Estimation, which employ random draws of the latent variables from their probability distributions. For illustration purposes, consider the use of simulated MLE for the model that we later present as Case Study 1. This is a binary choice (probit) model with 2 latent variables and six indicators (see the Case Study for further details). The likelihood function is as follows:

$$f_4(y, I \mid X; \alpha, \beta, \gamma, \Sigma) = \int\int_{Z^*} \Phi\{y(X\beta_1 + Z^*\beta_2)\} * \prod_{r=1}^{6} \frac{1}{\sigma_{\upsilon_r}} \phi\left[\frac{I_r - Z^*\alpha_r}{\sigma_{\upsilon_r}}\right] * \prod_{l=1}^{2} \frac{1}{\sigma_{\eta_l}} \phi\left[\frac{Z_l^* - X\gamma_l}{\sigma_{\eta_l}}\right] dZ^*$$

Note that since this is only a double integral, it is actually more efficient to estimate the model using numerical integration (as we do in the case study). However, the model serves well for illustration purposes.

Typically, the random draws are taken from a *N(0,I)* distribution, so we transform the likelihood by substituting:

$$Z_l^* = Xg_l + h_l \ , \ \ l = 1{,}2 \ , \ \ h \sim N(0, \Sigma_h \text{ diagonal}) \qquad \{\text{the structural LV equation}\}$$
$$\eta_l = \sigma_{\eta_l} \tilde{\eta}_l \text{, where } \tilde{\eta}_l \sim N(0{,}1)$$

which leads to:

$$f_4(y, I | X; \alpha, \beta, \gamma, \Sigma) = \int\int_{\tilde{\eta}} \Phi\{y(X\beta_1 + (X\gamma_1 + \sigma_{\eta_1}\tilde{\eta}_1)\beta_{12} + (X\gamma_2 + \sigma_{\eta_2}\tilde{\eta}_2)\beta_{22})\} *$$
$$\prod_{r=1}^{6} \frac{1}{s_{u_r}} f\left[\frac{I_r - (Xg_1 + s_{h_1}\tilde{h}_1)a_{1r} - (Xg_2 + s_{h_2}\tilde{h}_2)a_{2r}}{s_{u_r}}\right] * \prod_{l=1}^{2} f\left(\tilde{h}_l\right) d\tilde{h}$$

To simulate the likelihood, we take D random draws from $\tilde{\eta}_1$ and $\tilde{h}_2$ for each observation in the sample, denoted $\tilde{\eta}_1^d$ and $\tilde{\eta}_2^d$, *d=1,…,D*. The following is then an unbiased simulator for $f(y, I | X; \alpha, \beta, \gamma, \Sigma)$:

$$\tilde{f}_4(y, I | X; \alpha, \beta, \gamma, \Sigma) = \frac{1}{D}\sum_{d=1}^{D}\Big\{\Phi\{y(X\beta_1 + (X\gamma_1 + \sigma_{\eta_1}\tilde{\eta}_1^d)\beta_{12} + (X\gamma_2 + \sigma_{\eta_2}\tilde{\eta}_2^d)\beta_{22})\}$$
$$* \prod_{r=1}^{6} \frac{1}{\sigma_{\upsilon_r}} \phi\left[\frac{I_r - (X\gamma_1 + \sigma_{\eta_1}\tilde{\eta}_1^d)\alpha_{1r} - (X\gamma_2 + \sigma_{\eta_2}\tilde{\eta}_2^d)\alpha_{2r}}{\sigma_{\upsilon_r}}\right]\Big\}$$

The parameters are estimated by maximizing the simulated likelihood:

$$\max_{\alpha,\beta,\gamma,\Sigma} \sum_{n=1}^{N} \ln \tilde{f}_4\left(y_n, I_n | X_n; \alpha, \beta, \gamma, \Sigma\right)$$

Note that, by Jensen's Inequality, $\ln \tilde{f}_4$ is a biased estimator of $\ln f$. When a small number of draws is employed, this results in a non-negligible bias in the parameter estimates. Therefore, one has to verify that a sufficient number of draws is used to reduce this bias. This is usually done by estimating the model using various number of draws, and showing empirically that the parameter estimates are stable over a certain number of draws.

For more information on simulation methods for estimating discrete choice models, see McFadden (1986b and 1989) and Gourieroux and Monfort (1996).

Model Application

The measurement equations are used in estimation to provide identification of the latent constructs and further precision in the parameters estimates for the structural equations. For forecasting, we are interested in predicting the probability of the choice indicator,

$P(y|X;\alpha,\beta,\gamma,\Sigma)$. Furthermore, we do not have forecasts of the indicators, *I*. Therefore, the likelihood must be integrated over the indicators, and the model structure used for application is:

$$P(y|X;\alpha,\beta,\gamma,\Sigma) = \int_{X^*} P(y|X,X^*;\beta,\Sigma_\varepsilon) f_1(X^*|X;\gamma,\Sigma_\eta) dX^* \qquad (6)$$

So once the model is estimated, equation (6) can be used for forecasting and there is no need for latent variable measurement models nor the indicators.

Behavioral Framework for Choice Models with Latent Variables

The behavioral framework for choice models with latent variables is presented in Figure 6 [Ben-Akiva and Boccara, 1987]. The modeling framework presented here attempts to analyze explicitly latent psychological factors in order to gain information on aspects of individual behavior that cannot be inferred from market behavior or revealed preferences. In this behavioral framework, three types of latent factors are identified: attitudes, perceptions, and preferences.

The Cause-Effect Behavioral Relationships

Attitudes and perceptions of individuals are hypothesized to be key factors that characterize the underlying behavior. The observable explanatory variables, including characteristics of the individual (e.g., socio-economics, demographics, experience, expertise, etc.) and the attributes of alternatives (e.g., price) are linked to the individual's attitudes and perceptions through a causal mapping. Since attitudes and perceptions are unobservable to the analyst, they are represented by latent constructs. These latent attitudes and perceptions, as well as the observable explanatory variables, affect individuals' preferences toward different alternatives and their decision-making process.

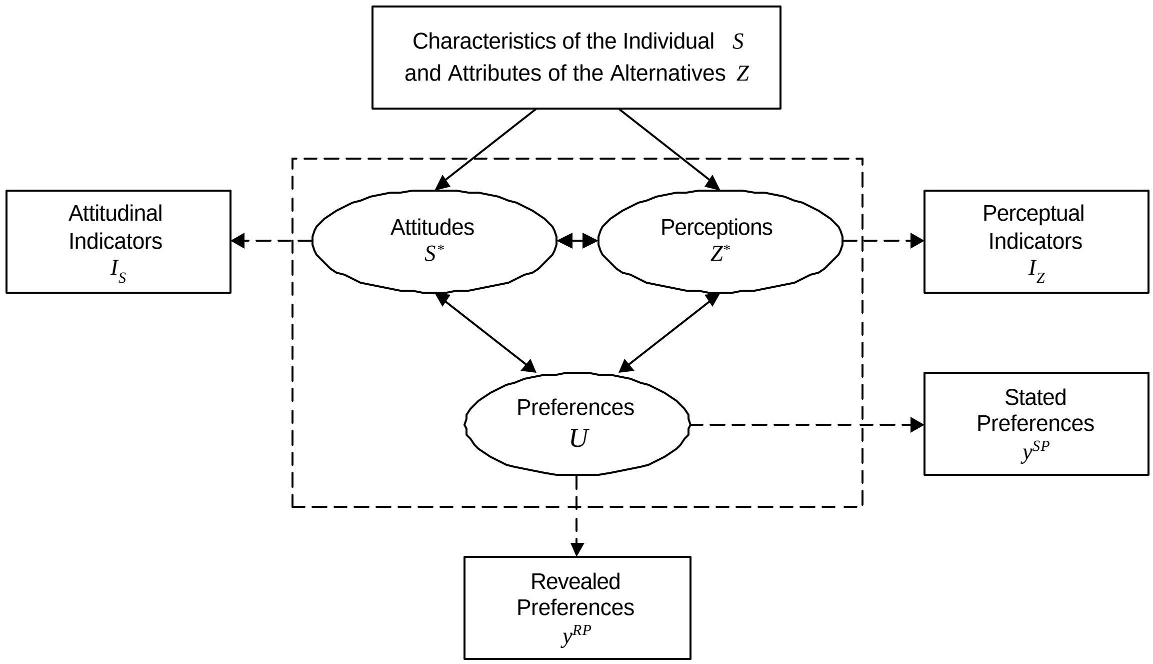

Figure 6: Behavioral Framework for Choice Models with Latent Variables

Perceptions are the individuals' beliefs or estimates of the levels of attributes of the alternatives. The choice process is expected to be based on perceived levels of attributes. Perceptions explain part of the random component of the utility function through individual-specific unobserved attributes. Examples of perceptions in a travel mode choice context for the transit alternative are *safety*, *convenience*, *reliability*, and *environmental friendliness*. Examples of perceptions for toothpaste are *health benefit* and *cosmetic benefit* (Elrod, 1998).

Attitudes are latent variables corresponding to the characteristics of the decision-maker. Attitudes reflect individuals' needs, values, tastes, and capabilities. They are formed over time and are affected by experience and external factors that include socioeconomic characteristics. Attitudes explain unobserved individual heterogeneity, such as taste variations, choice set heterogeneity and decision protocol heterogeneity. Examples of attitudes in a travel mode choice context are *the importance of reliability* or *preferences for a specific mode*. Examples of attitudes about toothpaste are the *importance of health benefits*, *cosmetic benefits*, and *price*.

In this framework, as in traditional random utility models, the individual's *preferences* are assumed to be latent variables. Preferences represent the desirability of alternative choices. These preferences are translated to decisions via a decision-making *process*. The process by which one makes a decision may vary across different decision problems or tasks, and is impacted by type of task, context, and socioeconomic factors (Gärling and Friman, 1998). Frequently, choice models assume a utility maximization decision process (as we do in our case studies). However, numerous other decision processes may be appropriate given the context, for example habitual, dominant attribute, or a series of decisions each with a different decision-making process. This framework is flexible and can incorporate various types of decision processes.

The Measurement Relationships

The actual market behavior or revealed preference (RP) and the preferences elicited in stated preference (SP) experiments are *manifestations* of the underlying preferences, and thus serve as indicators. Similarly, we may also have indicators for attitudes and perceptions such as responses to attitudinal and perceptual questions in surveys. For example, one could use rankings of the importance of attributes or levels of satisfaction on a semantic scale. As stated earlier, indicators are helpful in model identification and increase the efficiency of the estimated choice model parameters.

Benefits of the Framework

The integrated choice and latent variable modeling framework allows us to explicitly model the cognitive processes enclosed by the dashed lines in Figure 6. Incorporating such latent qualitative variables in choice models requires a hypothesis of the type and the role of the latent variables, as well as indicators of the latent variables.

The simple framework shown in Figure 6 is a bit deceiving. *Attitudes* can in fact be any latent characteristic of a decision-maker and thus incorporate concepts such as memory, awareness, tastes, goals, etc. Attitudes can be specified to have a causal relationship with other attitudes and perceptions, and vice-versa. Temporal variables can also be introduced in the specification, and different *processes* by which people make decisions could be included, such as those described in the section above. There is still a tremendous gap between descriptive behavioral theory and the ability of statistical models to reflect these behavioral hypotheses. Examining the choice process within this framework of latent characteristics and perceptions opens the door in terms of the types of behavioral complexities we can hope to capture, and can work to close the gap between these fields.

As with all statistical models, the consequences of mis-specification can be severe. Measurement error and/or exclusion of important explanatory variables in a choice model may result in inconsistent estimates of all parameters. As with an observable explanatory variable, excluding an important attitude or perception will also result in inconsistent estimates. The severity depends highly on the model at hand and the particular specification error, and it is not possible to make generalizations. Before applying the integrated choice and latent variable methodology, the decision process of the choice of interest must also be considered. For more information on behavioral decision theory, see Engel et al. (1973), Olson (1993), and other references listed in the "Supporting Research" section of this paper.

Case Studies

The unique features of the integrated choice modeling framework are demonstrated in three case studies. For each case study, the problem context, a problem-specific modeling framework, survey questions, model equations, and results are presented.

The Role of the Case Studies

These case studies have been assembled from a decade of research investigating the incorporation of attitudes and perceptions in choice modeling. The case studies provide conceptual examples of model frameworks, along with some specific equations, estimation results, and comparison of these models with standard choice models. The aim is to show that the methodology is practical, and to provide concrete examples. The case studies emphasize the general nature of the approach by providing likelihood functions for a variety of model structures, including the use of both SP and RP data, the introduction of an agent effect, and the use of logit, probit, and ordered probit.

Model Estimation

The dimensionalities of the likelihoods in each of the three case studies were small enough such that numerical integration was feasible and preferred over simultaneous estimation techniques. Therefore, numerical integration was used in all three studies. The first case study was estimated sequentially, where the choice probability was integrated over the latent variables in the second stage, resulting in consistent, inefficient estimates of the parameters. In the second and third case studies, the latent variable and choice models were estimated jointly (by programming the likelihood function in *GAUSS* and employing its numerical integration routines), resulting in consistent, efficient estimates. Identification was determined via application of the *Three-step Rule* as described earlier, as well as using the inverse Hessian to check for local identification at the solution point.

Further References

Additional applications of the integrated approach can be found in Boccara (1989), Morikawa (1989), Gopinath (1994), Bernardino (1996), Börsch-Supan et al. (1996), Morikawa et al. (1996), and Polydoropoulou (1997).

Case Study 1: Mode Choice with Latent Attributes

The first case study (Morikawa, Ben-Akiva, and McFadden, 1996) presents the incorporation of the latent constructs of convenience and comfort in a mode choice model. The model uses data collected in 1987 for the Netherlands Railways to assess factors that influence the choice between rail and car for intercity travel. The data contain revealed choices between rail and auto for an intercity trip. In addition to revealed choices, the data also include subjective evaluation of trip attributes for both the chosen and unchosen modes, which were obtained by asking questions such as those shown in Table 1. The resulting subjective ratings are used as indicators for latent attributes. It is presumed that relatively few latent variables may underlie the resulting ratings data, and two latent variables, *ride comfort* and *convenience*, were identified through exploratory factor analysis.

Figure 7 presents the framework for the mode choice model. The revealed choice is used as an indicator of utility, and the attribute ratings are used as indicators for the two latent variables. Characteristics of the individual and observed attributes of the alternative modes are exogenous explanatory variables. Figure 8 provides a full path diagram of the model, noting the relationships between each variable.

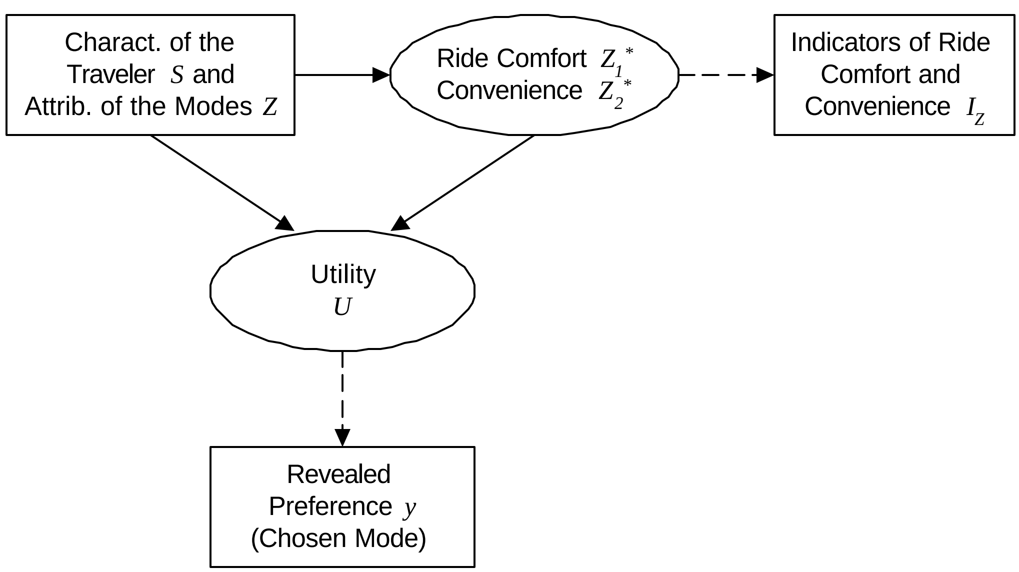

Figure 7: Modeling Framework for Mode Choice with Latent Attributes

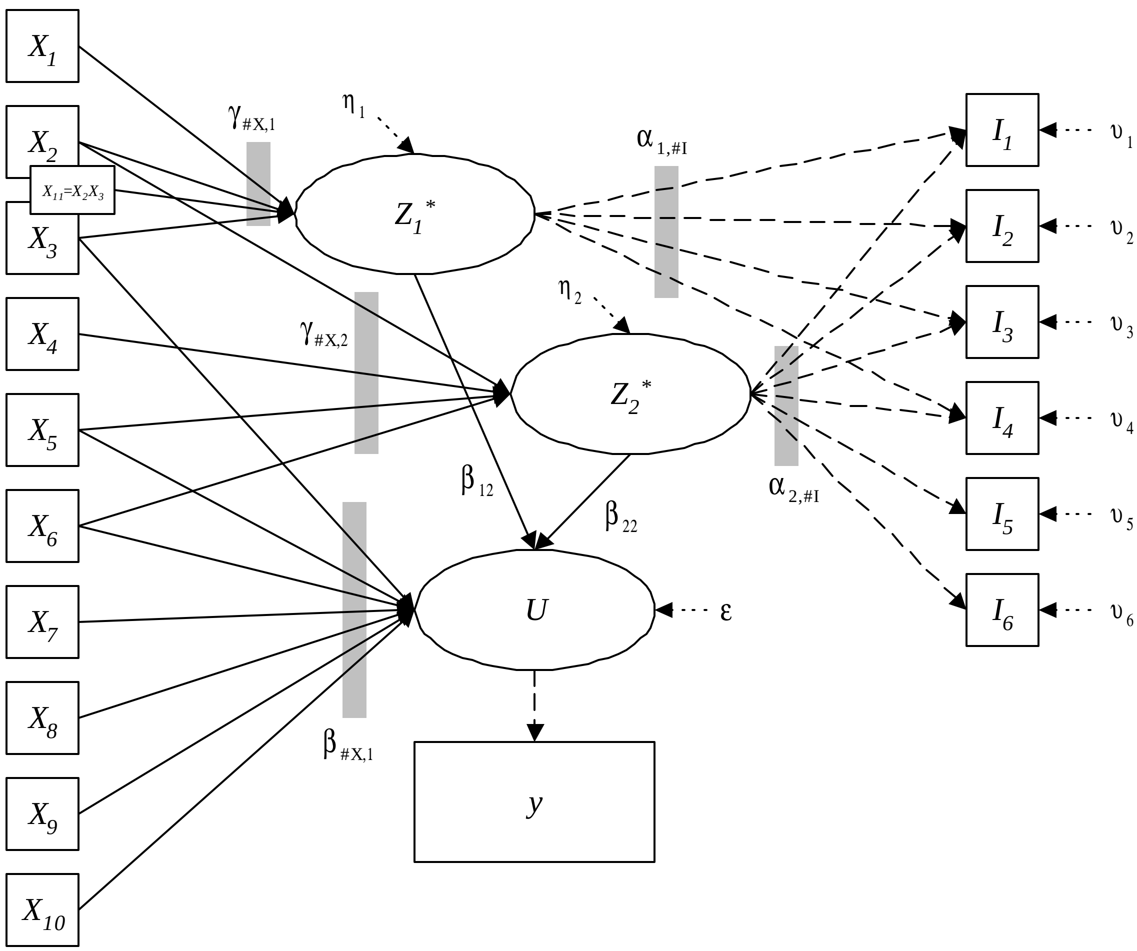

Figure 8: Full Path Diagram for Mode Choice Model with Latent Attributes (See Table 2 and the model equations for notation.)

Table 1: Indicators for Ride Comfort and Convenience

Please rate the following aspects for the auto trip:

	very…………very poor………..good				
Relaxation during the trip	1	2	3	4	5
Reliability of the arrival time	1	2	3	4	5
Flexibility of choosing departure time	1	2	3	4	5
Ease of traveling with children and/or heavy baggage	1	2	3	4	5
Safety during the trip	1	2	3	4	5
Overall rating of the mode	1	2	3	4	5

The mode choice model with latent attributes is specified by the following equations. All variables, including the latent variables, are measured in terms of the difference between rail and auto. This was done to reduce the dimensionality of the integral (from 4 to 2), and was not necessary for identification of the joint choice/latent variable model.

Structural Model

$$\underset{(1X1)}{Z_l^*} = \underset{(1X10)(10X1)}{Xg_l} + \underset{(1X1)}{h_l}\ ,\quad l = 1,2\ ,\quad h \sim N(0, \Sigma_h \text{ diagonal}) \qquad \{2 \text{ equations}\}$$

$$\underset{(1X1)}{U} = \underset{(1X10)(10X1)}{Xb_1} + \underset{(1X2)(2X1)}{Z^* b_2} + \underset{(1X1)}{e}\ ,\quad e \sim N(0,1) \qquad \{1 \text{ equation}\}$$

Measurement Model

$$\underset{(1X1)}{I_r} = \underset{(1X2)(2X1)}{Z^* a_r} + \underset{(1X1)}{u_r}\ ,\quad r = 1,\ldots,6\ ,\quad u \sim N(0, \Sigma_u \text{ diagonal}) \qquad \{6 \text{ equations}\}$$

$$\underset{(1X1)}{y} = \begin{cases} 1, & \text{if } U > 0 \\ -1, & \text{if } \underset{(1X1)}{U} \le 0 \end{cases} \qquad \{1 \text{ equation}\}$$

Note that the covariances of the error terms in the latent variable structural and measurement model are constrained to be equal to zero (denoted by the "*S diagonal*" notation).

Likelihood function

$$f(y, I | X; a, b, g, \Sigma) = \int\int_{Z^*} \Phi\{y(Xb_1 + Z^* b_2)\} *$$
$$\prod_{r=1}^{6} \frac{1}{\sigma_{\upsilon_r}} \phi\left[\frac{I_r - Z^*\alpha_r}{\sigma_{\upsilon_r}}\right] * \prod_{l=1}^{2} \frac{1}{\sigma_{\eta_l}} \phi\left[\frac{Z_l^* - X\gamma_l}{\sigma_{\eta_l}}\right] dZ^*$$

Results

The parameters to be estimated include: β (9 parameters estimated), α (8 parameters estimated, 2 parameters constrained to one for identification, 2 parameters constrained to zero based on exploratory factor analysis), γ (8 parameters estimated), and the standard deviations σ_υ (6 parameters) and σ_η (2 parameters), where the covariances of the latent variable equations are restricted to zero. Unless otherwise noted, parameters set to zero were done based on statistical tests and a priori hypothesis about the behavior. All parameters except the variances are reported.

The results are shown in Table 2. Estimation was done via sequential numerical integration: first the latent variable model was estimated, then the choice model (including integration over the latent variable) was estimated. The dataset included 219 observations. The top panel displays the estimation results of two different choice models: the second column is the choice model *without* the latent variables, and the first column is the choice model *with* the latent variables. The integrated choice and latent variable model consists of the choice model with latent variables (the first column of the upper panel) and the latent variable model (displayed in the lower panel of Table 2). The table for the latent variable model displays the estimation results of both the structural and measurement equations for each of the two latent variables *comfort* (the first column) and *convenience* (the second column). The latent variable model is made up of many equations: one structural equation for comfort, one structural equation for convenience, and six measurement equations for comfort and convenience.

Both of the latent attributes have significant parameter estimates. Inclusion of the latent attributes identified by the linear structural equation resulted in a large improvement in the goodness-of-fit of the discrete choice model. The rho-bar-squared for the model with latent attributes uses a degree-of-freedom correction involving two variables beyond those used in the model without latent variables, and thus this degree of freedom adjustment only accounts for the estimated parameters of the choice model.

While the indicators used for comfort and convenience in this case study are adequate, the structural equations are not particularly strong because the available explanatory variables of comfort and convenience were limited. In general, it can be difficult to find causes for the latent variables. This issue needs to be thoroughly addressed in the data collection phase.

Table 2: Estimation Results of Mode Choice Model with Latent Attributes

CHOICE MODEL

		WITH Latent Attributes		WITHOUT Latent Attributes	
	Explanatory Variables	Est. β	t-stat	Est. β	t-stat
X10	Rail constant	0.32	1.00	0.58	2.00
X9	Cost per person	-0.03	-4.10	-0.03	-4.20
X3	Line-haul time	0.08	0.20	-0.41	-1.60
X6	Terminal time	-1.18	-2.60	-1.57	-4.20
X5	Number of transfers	-0.32	-1.70	-0.20	-1.30
X8	Business trip dummy	1.33	3.60	0.94	3.60
X7	Female dummy	0.65	2.60	0.47	2.30
*Z1**	Ride comfort (latent)	0.88	2.70	------	------
*Z2**	Convenience (latent)	1.39	4.10	------	------
	Rho-bar-Squared	0.352		0.242	

LATENT VARIABLE MODEL

	Structural Model	Comfort *Z1**		Convenience *Z2**	
	(2 equations total, 1 per column)	Est. γ1	t-stat	Est. γ2	t-stat
X2	Age >40	-0.23	-1.40	0.41	3.30
X1	First class rail rider	0.29	1.00	------	------
X3	Line haul travel time (rail-auto)	-0.29	-1.30	------	------
X6	Terminal time (rail-auto)	------	------	-0.52	-2.10
X5	Number of transfers by rail	------	------	-0.05	-0.60
X4	Availability of free parking for auto	------	------	0.16	1.60
X11	(Age >40) * (Line haul travel time)	-0.04	-0.10	------	------

	Measurement Model	Comfort *Z1**		Convenience *Z2**	
	(6 equations total, one per row)	Est. α1	t-stat	Est. α2	t-stat
I1	Relaxation during trip	1.00	------	0.17	0.80
I2	Reliability of the arrival time	0.77	1.80	1.00	------
I5	Flexibility of choosing departure time	------	------	1.49	4.30
I6	Ease of traveling with children/baggage	------	------	1.16	1.16
I3	Safety during the trip	0.69	3.10	0.33	2.00
I4	Overall rating of the mode	1.64	2.60	2.43	5.90

Case Study 2: Employees' Adoption of Telecommuting

The second case study (Bernardino, 1996) assesses the potential for the adoption of telecommuting by employees. Figure 9 presents the modeling framework. The behavioral hypothesis is that an employee faced with a telecommuting arrangement will assess the impact of the arrangement on lifestyle, work-related costs and income, and then decide whether to adopt telecommuting. Telecommuting is expected to influence lifestyle quality by providing the employee with the benefit of increased flexibility to adjust work schedule, work load, personal needs, and commuting patterns. The perceived impact is expected to vary according to the characteristics of the individual and of the program. Telecommuting is also expected to impact household expenditures, such as utilities, equipment, day care, and transportation. Figure 10 provides a full path diagram of the model, noting the relationships between each variable.

The employee's decision to adopt a telecommuting program in a simulated choice experiment is modeled as a function of her/his motivations and constraints, as well as the impacts of the available program on lifestyle quality, work-related costs, and income. Changes in income are included in the telecommuting scenarios, while latent constructs of benefit (i.e. enhancement to lifestyle quality) and cost are estimated. To obtain indicators for benefit, respondents are asked to rate the potential benefits of the telecommuting program on a scale from 1 to 9 as shown in Table 3. These responses provide indicators for the latent variable model. The latent cost variable is manifested by the employees' responses to questions about the expected change in home office costs, child and elder care costs, and overall work-related costs as shown in Table 4. The employee is assumed to have a utility maximization behavior, and thus will choose to adopt a particular telecommuting option if the expected change in utility is positive. This decision is influenced by the characteristics of the arrangement, the individual's characteristics and situational constraints, and the perceived benefits and costs of the arrangement.

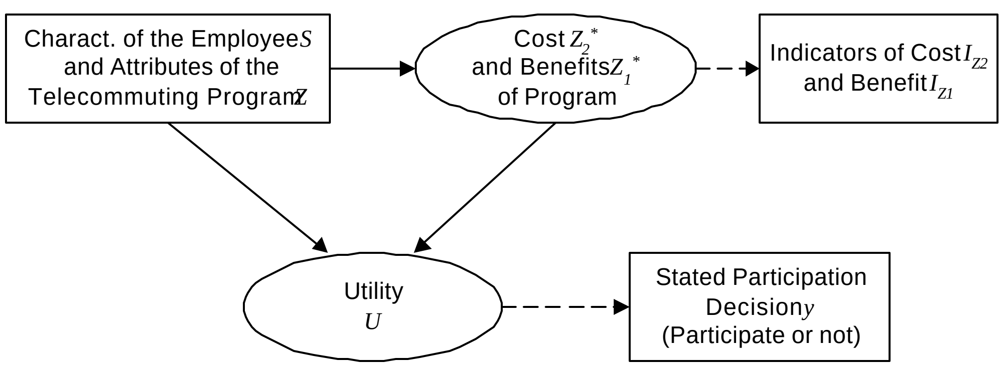

Figure 9:
Modeling Framework for Employee's Adoption of Telecommuting

The adoption of telecommuting model is specified by the following equations.

Structural Model

$$\underset{(1X1)}{Z_l^*} = \underset{(1X14)(14X1)}{X g_l} + \underset{(1X1)}{h_l} \ , \ \ l = 1,2 \ , \ \ h \sim N(0, \Sigma_h \text{ diagonal}) \qquad \{2 \text{ equations}\}$$

$$\underset{(1X1)}{U} = \underset{(1X14)(14X1)}{X b_1} + \underset{(1X2)\,(2X1)}{Z^* b_2} + \underset{(1X1)}{e} \ , \ \ e \sim \text{standard logistic} \qquad \{1 \text{ equation}\}$$

Measurement Model

$$\underset{(1X1)}{I_r} = \underset{(1X2)(2X1)}{Z^* \alpha_r} + \underset{(1X1)}{\upsilon_r} \ , \ \ r = 1,\ldots,14 \ , \ \ u \sim N(0, \Sigma_u \text{ diagonal}) \qquad \{14 \text{ equations}\}$$

$$\underset{(1X1)}{y} = \underset{(1X1)}{\begin{cases} 1, & \text{if } U > 0 \\ -1, & \text{if } U \le 0 \end{cases}} \qquad \{1 \text{ equation}\}$$

Likelihood Function

$$f(y, I \mid X; a, b, g, \Sigma) = \int\!\!\int_{Z^*} \left(\frac{1}{1 + \exp^{-(Xb_1 + Z^* b_2) y}} \right) * \prod_{r=1}^{R} \frac{1}{\sigma_{\upsilon_r}} \phi \left[\frac{I_r - Z^* \alpha_r}{\sigma_{\upsilon_r}} \right] *$$

$$\prod_{l=1}^{2} \frac{1}{s_{h_l}} f \left[\frac{Z_l^* - X g_l}{s_{h_l}} \right] dZ^*$$

Table 3: Indicators of Benefit

What type of impact would you expect the telecommuting arrangement to have on:

	extremely…………………								extremely
	negative……………………								.positive
Your schedule flexibility	1	2	3	4	5	6	7	8	9
Your productivity	1	2	3	4	5	6	7	8	9
Your autonomy in your job	1	2	3	4	5	6	7	8	9
The productivity of the group you work with	1	2	3	4	5	6	7	8	9
Your family life	1	2	3	4	5	6	7	8	9
Your social life	1	2	3	4	5	6	7	8	9
Your job security	1	2	3	4	5	6	7	8	9
Your opportunity for promotion	1	2	3	4	5	6	7	8	9
Your sense of well being	1	2	3	4	5	6	7	8	9
Your job satisfaction	1	2	3	4	5	6	7	8	9
Your life, overall	1	2	3	4	5	6	7	8	9

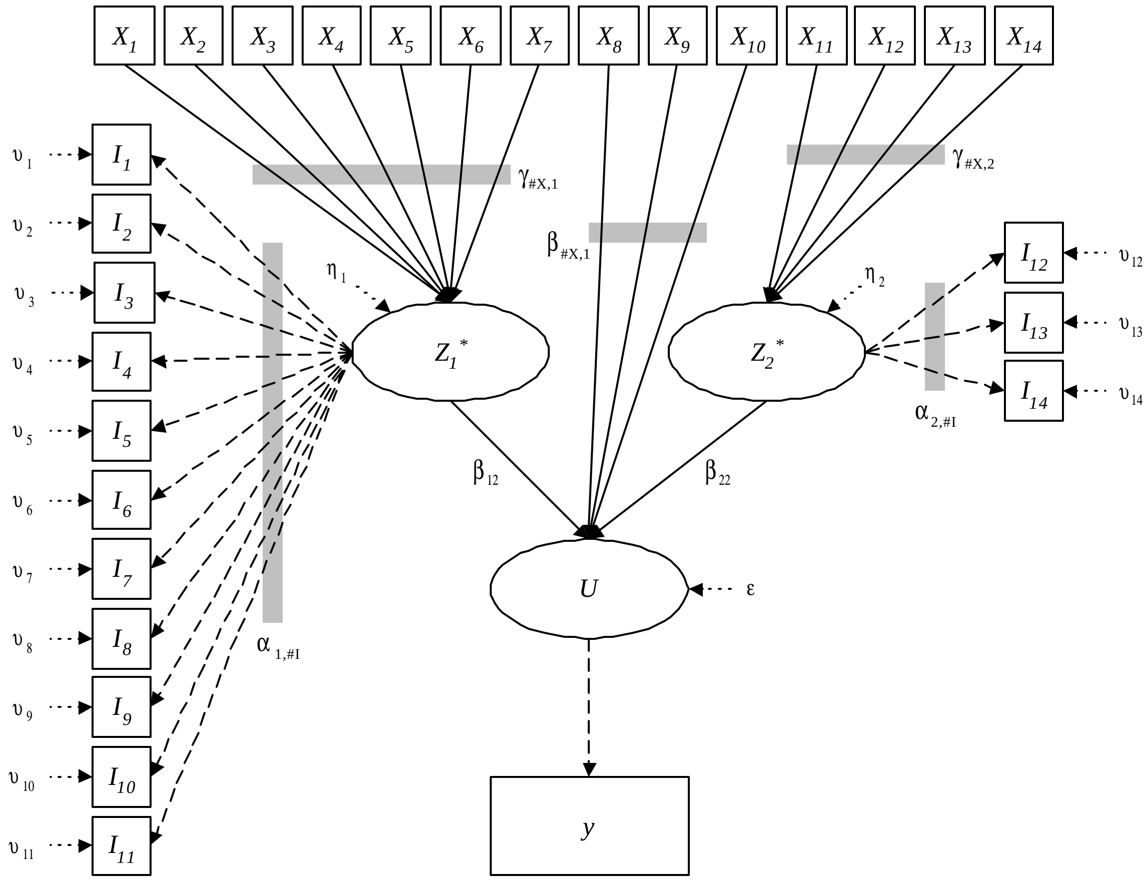

Figure 10:
Full Path Diagram for Model of Employee's Adoption of Telecommuting
(See Table 5 and the model equations for notation.)

Table 4: Indicators of Cost

How would you expect the telecommuting arrangement to impact your expenditures on:

home utilities:	( ) decrease	( ) remain the same	( ) increase
child care:	( ) decrease	( ) remain the same	( ) increase
elder care:	( ) decrease	( ) remain the same	( ) increase
overall working costs:	( ) decrease	( ) remain the same	( ) increase

Results

The parameters to be estimated include: β (5 parameters estimated), α (13 parameters estimated, 1 parameter constrained to one for identification), γ (11 parameters estimated), and the standard deviations σ_υ (14 parameters) and σ_η (2 parameters), where the covariances of the latent variable equations are restricted to zero. Unless otherwise noted, parameters set to zero were done based on statistical tests and a priori hypothesis about the behavior.

The estimation results are shown in Table 5 (estimated variances of the error terms are not reported). The model was estimated using observations from 440 individuals and employed a simultaneous numerical integration estimation procedure. The top panel displays the results of the choice model, which includes the latent explanatory variables benefit and cost. The lower panel displays the results for the latent variable model. The latent variable model consists of many equations: a structural equation for benefit, a structural equation for cost, 11 measurement equations for benefit (one equation per row), and 3 measurement equations for cost (again, one equation per row).

This model of the employee's adoption decision contains more information and allows for a clearer behavioral interpretation than standard choice models. It demonstrates the impact of different telecommuting arrangements on the employee's lifestyle and work-related costs, as a function of the employee's characteristics and situational constraints. The results indicate that females and employees with young children perceive a higher beneficial impact from telecommuting on lifestyle quality than their counterparts. Note that unlike the other two case studies presented in this paper, a survey was conducted that was designed specifically for this model, and, as a result, the structural models are quite strong with solid causal variables. For more information on these models and other models for telecommuting behavior, see Bernardino (1996).

Table 5: Estimation Results of a Telecommuting Choice Model with Latent Attributes

CHOICE MODEL

	Explanatory Variables	Est. β	t-stat
X8	Telecommuting specific constant	2.02	8.94
X9	Higher salary to telecommuters (relative to 'same')	0.50	1.12
X10	Lower salary to telecommuters (relative to 'same')	-2.36	-5.78
*Z1**	Benefit (latent variable)	0.99	7.01
*Z2**	Cost (latent variable)	-0.37	-3.12
	Rho-bar-Squared	0.35	

LATENT VARIABLE MODEL

	*Structural Model for Benefits Z1** (1 equation)	Est. γ1	t-stat
X1	Min # of telecommuting days/week	-0.15	-6.65
X2	Max # of telecommuting days/week * team structure dummy	0.10	3.02
X3	Max # telecommuting days/week * individual structure dummy	-0.04	-1.99
X4	Telework center telecommuting dummy	-1.02	-14.75
X5	Travel time * female dummy	0.69	7.47
X6	Travel time * male dummy	0.27	3.21
X7	Child under 6 years old in household dummy	0.55	7.46
	Squared multiple correlation for structural equation	0.28	

	*Measurement Model for Benefits Z1** (11 equations)	Est. α1	t-stat
I1	Social life	0.59	11.61
I2	Family life	0.80	18.37
I3	Opportunity for job promotion	0.32	6.19
I4	Job security	0.41	8.15
I5	Schedule flexibility	0.76	14.40
I6	Job autonomy	0.60	12.51
I7	Your Productivity	0.92	20.87
I8	Group productivity	0.61	12.43
I9	Sense of well being	1.04	24.86
I10	Job satisfaction	1.07	24.84
I11	Life overall	1.00	-----

	*Structural Model for Cost Z2** (1 equation)	Est. γ2	t-stat
X11	Day care costs proxy	0.39	2.00
X12	Home office utilities proxy	-0.36	-2.70
X13	Equipment costs	0.76	2.50
X14	Weekly transportation costs	0.65	2.91
	Squared multiple correlation for structural equation	0.21	

	*Measurement Model for Cost Z2** (3 equations)	Est. α2	t-stat
I12	Day care costs	0.37	4.78
I13	Home office utilities costs	-0.11	-3.07
I14	Overall working costs	0.50	3.63

Case Study 3: Usage of Traffic Information Systems

The objective of the third case study (Polydoropoulou, 1997) is to estimate the willingness to pay for Advanced Traveler Information Systems. The model uses data collected for the SmarTraveler test market in the Boston area. SmarTraveler is a service that provides real-time, location-specific, multi-modal information to travelers via telephone.

Figure 11 shows the framework for the model, which includes a latent variable of satisfaction as an explanatory variable in the usage decision. Travelers' satisfaction ratings of SmarTraveler are used as indicators of the satisfaction latent construct. Table 6 shows the survey questions used to obtain ratings of satisfaction. The model assumes that each traveler has an underlying utility for the SmarTraveler service. The utility is a function of the service attributes such as cost and method of payment, as well as the overall satisfaction with the service. Since utility is not directly observable, it is a latent variable, and the responses to the alternate pricing scenarios serve as indicators of utility. Respondents were presented with several pricing scenarios, and then asked what their usage rate (in terms of number of calls per week) or likelihood of subscribing to the service would be under each scenario. Two types of scenarios were presented: a measured pricing structure in which travelers are charged on a per call basis (corresponds to SP1 responses) and a flat rate pricing structure in which travelers pay a monthly subscription fee (corresponds to SP2 responses). Travelers' revealed preference for free service is reflected by the actual usage rate, which serves as an additional indicator of utility. Figure 12 provides a full path diagram of the model, noting the relationships between each variable in the model.

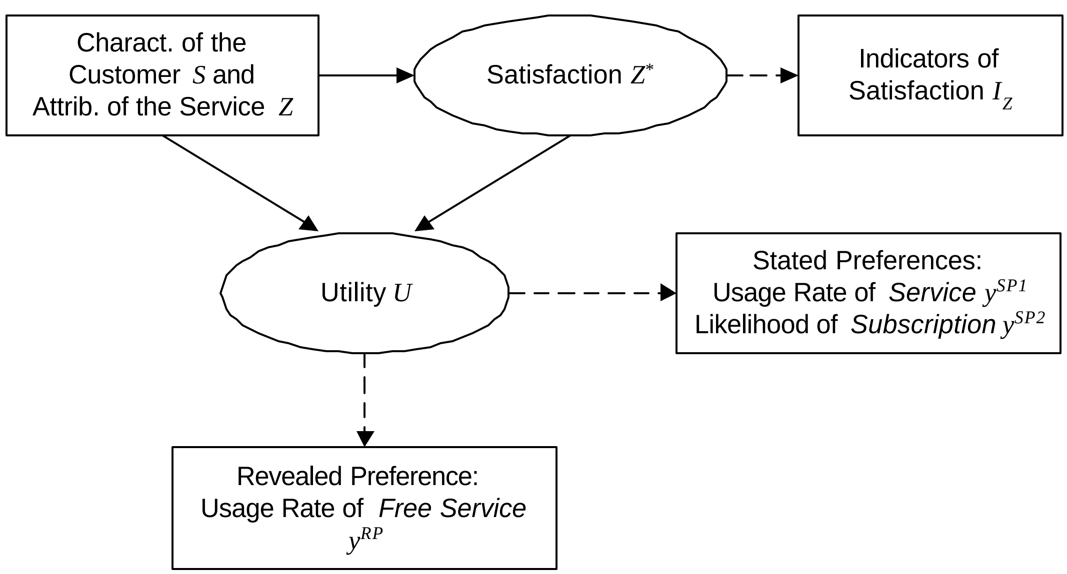

Figure 11:
Modeling Framework for Usage of SmarTraveler

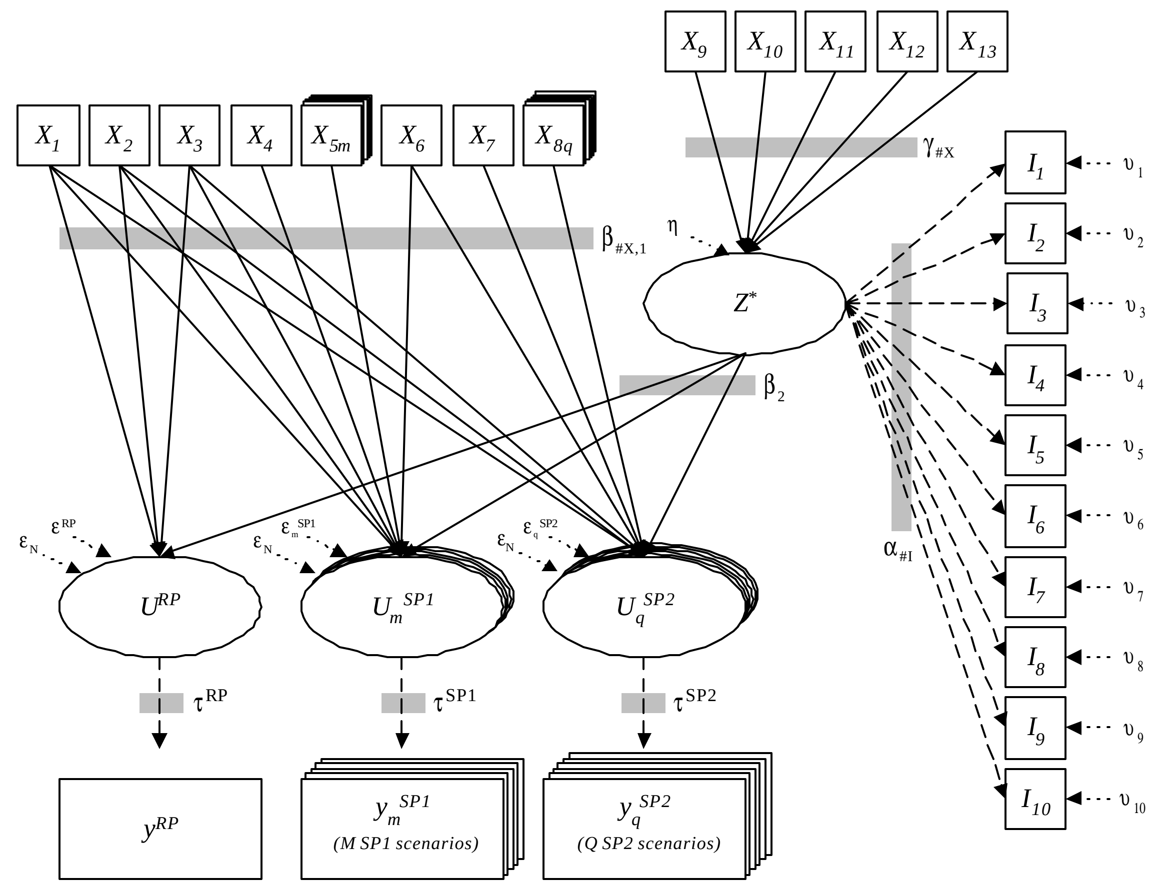

Figure 12:
Full Path Diagram for Model of Usage of SmarTraveler
(See Table 7 and model equations for notation.)

Table 6: Indicators of Satisfaction with SmarTraveler Service

Please rate your level of satisfaction with the following aspects of the existing SmarTraveler service.

	extremely dissatisfied								extremely satisfied
Ease of use	1	2	3	4	5	6	7	8	9
Up to the minute information	1	2	3	4	5	6	7	8	9
Availability on demand	1	2	3	4	5	6	7	8	9
Accuracy of information	1	2	3	4	5	6	7	8	9
Level of detail of information	1	2	3	4	5	6	7	8	9
Provision of alternate routes	1	2	3	4	5	6	7	8	9
Hours of operation	1	2	3	4	5	6	7	8	9
Coverage of major routes	1	2	3	4	5	6	7	8	9
Cost of service	1	2	3	4	5	6	7	8	9
Overall satisfaction with service	1	2	3	4	5	6	7	8	9

All of the choice variables are ordered categorical. The revealed preference choice (y^{RP}) and the stated usage rate (y^{SP1}) can take on the following values:

$$y = \begin{cases} 1, & \text{if less than 1 call per week} \\ 2, & \text{if 1 to 4 calls per week} \\ 3, & \text{if 5 to 9 calls per week} \\ 4, & \text{if more than 9 calls per week} \end{cases}$$

The stated likelihood of subscription (y^{SP2}) can take on the following values:

$$y = \begin{cases} 1, & \text{if very unlikely to subscribe} \\ 2, & \text{if somewhat unlikely to subscribe} \\ 3, & \text{if somewhat likely to subscribe} \\ 4, & \text{if very likely to subscribe} \end{cases}$$

The following equations specify the model of SmarTraveler usage.

Structural Model

$$\underset{(1X1)}{Z^*} = \underset{(1X13)}{X^{RP}}\underset{(13X1)}{g} + \underset{(1X1)}{h}\ ,\ h \sim N(0, s_h^2) \qquad \{1 \text{ equation}\}$$

Utility equations: {1+M+Q equations}

$$U^{RP} = V^{RP} + \tilde{e}^{RP} = X^{RP}b_1 + Z^*b_2 + e_N + e^{RP} \quad , \quad e^{RP} \sim N(0,1)$$

$$U_m^{SP1} = V_m^{SP1} + \tilde{e}_m^{SP1} = X_m^{SP1}b_1 + Z^*b_2 + e_N + e_m^{SP1} \quad , \quad e_m^{SP1} \sim N(0, s_{SP1}^2), \quad m = 1,\ldots,M$$

$$\underset{(1X1)}{U_q^{SP2}} = V_q^{SP2} + \tilde{e}_q^{SP2} = \underset{(1X13)}{X_q^{SP2}}\underset{(13X1)}{b_1} + \underset{(1X1)}{Z^*}\underset{(1X1)}{b_2} + \underset{(1X1)}{e_N} + \underset{(1X1)}{e_q^{SP2}} \quad , \quad e_q^{SP2} \sim N(0, s_{SP2}^2), \quad q = 1,\ldots,Q$$

where:

m denotes a particular measured rate scenario
q denotes a particular flat rate scenario.

The disturbance in the utility equations, $\tilde{e}$, are made up of 2 components: a respondent-specific component and a dataset/scenario specific component. The random disturbance characterizing each respondent, e_N , is constant for any respondent across pricing scenarios, and captures the correlation among responses from the same individual (an "agent effect"). The assumed distribution is $e_N \sim N(0, s_N^2)$.

Measurement Model

$$\underset{(1X1)}{I_r} = \underset{(1X1)}{Z^*}\underset{(1X1)}{a_r} + \underset{(1X1)}{u_r}\ ,\ r = 1,\ldots,10\ ,\ u \sim N(0,\Sigma_u \text{ diagonal}) \qquad \{10 \text{ equations}\}$$

$$y^{RP} = t,\ \text{if } t_{t\text{-}1}^{RP} < U^{RP} \le t_t^{RP} \qquad t = 1,\ldots,4$$
$$y_m^{SP1} = t,\ \text{if } t_{t\text{-}1}^{SP1} < U_m^{SP1} \le t_t^{SP1} \qquad t = 1,\ldots,4 \quad m = 1,\ldots,M$$
$$y_q^{SP2} = t,\ \text{if } t_{t\text{-}1}^{SP2} < U_q^{SP2} \le t_t^{SP2} \qquad t = 1,\ldots,4 \quad q = 1,\ldots,Q$$

t are unknown threshold parameters, with $t_0 = -\infty$, $\tau_1 = 0$ (for identification), $t_4 = \infty$

Additional Notation

$$y_t^{RP} = \begin{cases} 1, & \text{if } y^{RP} = t \\ 0, & \text{otherwise} \end{cases}$$
$$y_{mt}^{SP1} = \begin{cases} 1, & \text{if } y_m^{SP1} = t \\ 0, & \text{otherwise} \end{cases}$$
$$y_{qt}^{SP2} = \begin{cases} 1, & \text{if } y_q^{SP2} = t \\ 0, & \text{otherwise} \end{cases}$$

Likelihood Function

$$f(y, I | a, b, g, \Sigma, t) = \int\int \left[\sum_{t=1}^{4} y_t^{RP}\left[\Phi\left(\frac{t_t^{RP} - V^{RP} - e_N)}{1}\right) - \Phi\left(\frac{t_{t-1}^{RP} - V^{RP} - e_N}{1}\right)\right]\right] *$$
$$\left[\prod_{m=1}^{M}\left(\sum_{t=1}^{4} y_{mt}^{SP1}\left[\Phi\left(\frac{t_t^{SP1} - V_m^{SP1} - e_N}{s_{SP1}}\right) - \Phi\left(\frac{t_{t-1}^{SP1} - V_m^{SP1} - e_N}{s_{SP1}}\right)\right]\right)\right] *$$
$$\left[\prod_{q=1}^{Q}\left(\sum_{t=1}^{4} y_{qt}^{SP2}\left[\Phi\left(\frac{t_t^{SP2} - V_q^{SP2} - e_N}{s_{SP2}}\right) - \Phi\left(\frac{t_{t-1}^{SP2} - V_q^{SP2} - e_N}{s_{SP2}}\right)\right]\right)\right] *$$
$$\left[\prod_{r=1}^{10} \frac{1}{\sigma_{\upsilon_r}} \phi\left(\frac{I_r - Z^*\alpha_r}{\sigma_{\upsilon_r}}\right)\right] * \left[\frac{1}{\sigma_N}\phi\left(\frac{\varepsilon_N}{\sigma_N}\right)\right]\left[\frac{1}{\sigma_\eta}\phi\left(\frac{Z^* - X^{RP}\gamma}{\sigma_\eta}\right)\right] dZ^* d\varepsilon_N$$

Results

The parameters to be estimated include: β (9 parameters estimated), α (9 parameters estimated, 1 parameter constrained to one for identification), γ (5 parameters estimated), the threshold parameters τ, and the standard deviations σ_{υ} (10 parameters), σ_{η} (1 parameter), σ_{SP1} (1 parameter) σ_{SP2} (1 parameter), σ_{N} (1 parameter), where σ_{RP} was constrained to one for identification and the covariances of the latent variable equations are restricted to zero. Unless otherwise noted, parameters set to zero were done based on statistical tests and a priori hypothesis about the behavior.

Table 7 shows the estimation results for this model (estimated threshold parameters, τ, and variances of the error terms are not reported). The model was estimated using observations from 442 individuals, all of whom are SmarTraveler users, and a simultaneous numerical integration estimation procedure. Results of two choice models are presented: one without the satisfaction latent variable (the right column of the top panel) and one that includes the satisfaction latent variable (the left column of the top panel). The integrated choice and latent variable model consists of the choice model with the satisfaction variable and the latent variable model (one structural equation and 10 measurement equations).

We found that incorporation of satisfaction in the utility of SmarTraveler model significantly improved the goodness of fit of the choice model. The rho-bar-squared for the model with latent attributes uses a degree-of-freedom correction involving one variable (for the satisfaction latent variable) beyond those used in the model without the latent variable, and thus this degree of freedom adjustment only accounts for the estimated parameters of the choice model. See Polydoropoulou (1997) for additional model estimation results for this model, and for additional models of non-users and of other behavioral responses to SmarTraveler.

Table 7: Estimation Results of ATIS Usage Model with Latent Satisfaction

CHOICE MODEL

	Utility of SmarTraveler Service	WITH the Satisfaction Latent Variable		WITHOUT the Satisfaction Latent Variable	
	Explanatory Variables	Est. β	t-stat	Est. β	t-stat
X6	Constant for actual market behavior	0.94	5.20	0.97	5.90
X4	Constant for measured service	0.56	3.90	0.59	4.30
X7	Constant for flat rate service	0.10	0.70	0.11	0.80
X5	Price per call (cents/10)	-0.31	-15.90	-0.31	-15.80
X8	Subscription fee ($/10)	-1.29	-15.50	-1.27	-16.30
X1	Income: $30,000-$50,000	0.02	0.10	0.15	1.00
X2	Income: $50.001-$75.000	0.32	2.10	0.37	2.60
X3	Income: >$75.000	0.35	2.40	0.22	1.60
*Z**	Satisfaction Latent Variable	0.16	4.50	------	------
	Rho-bar-Squared	0.65		0.49	

LATENT VARIABLE MODEL

	Structural Model (1 equation)	Est. γ	t-stat
X9	Gender (male dummy)	-0.19	-2.40
X10	NYNEX user	-0.86	-10.50
X11	Cellular One user	-1.08	-8.20
X12	Age: 25-45 years	-0.26	-1.60
X13	Age: >45 years	-0.24	-1.40
	Squared multiple correlation for structural model	0.104	

	Measurement Model (10 equations)	Est. α	t-stat	R^2_{Ir}
I1	Ease of use	0.46	7.80	0.15
I2	Up to the minute information	1.26	21.60	0.64
I3	Availability on demand	0.47	8.2	0.18
I4	Accuracy of information	1.19	23.10	0.69
I5	Level of Detail of information	1.10	22.60	0.63
I6	Suggestions of alternative routes	0.75	7.80	0.16
I7	Hours of operation	0.57	7.40	0.13
I8	Coverage of major routes	0.59	12.60	0.25
I9	Cost of service	0.19	5.30	0.06
I10	Overall satisfaction with service	1.00	-----	0.82

Practical Findings from the Case Studies

In the case studies reported here, and in our other applications of the methodology, we generally find that implementation of the integrated choice and latent variable model framework results in:

- improvements in goodness of fit over choice models without latent variables.
- latent variables that are statistically significant in the choice model, with correct parameter signs
- a more satisfying behavioral representation

Several practical lessons were learned from our application of the methodology. First, in terms of the measurement equations (*eq. 3*), we found that a sufficient number of indicators relevant to the latent variable under consideration, as well as variability among the indicators, are critical success factors. Second, for the structural equations (*eq. 1*), we found that it can be difficult to find solid causal variables (X) for the latent variables. In some cases, it is difficult to even conceptually define good causal variables, that is, cases in which there are no good socioeconomic characteristics or observable attributes of the alternatives that sufficiently explain the latent attitudes and/or perceptions. However, quite frequently, even if one can define good causal variables, these types of data have not been collected and are not included in the dataset. To address both of these issues, it is critical for the successful application of this methodology, to first think clearly about the behavioral hypotheses behind the choices, then develop the framework, and *then* design a survey to support the model. The final major lesson is that these integrated models require both customized programs and fast computers for estimation. The estimation programs and models tend to be complex, and therefore the use of synthetic data to confirm the program's ability to reproduce the parameters should be done as a matter of routine. Such a test provides assurance that the model is identified and that the likelihood is programmed correctly, but does not otherwise validate the model specification.

Conclusion

In this paper, we present a general methodology and framework for including latent variables—in particular, attitudes and perceptions—in choice models. The methodology provides a framework for the use of psychometric data to explicitly model attitudes and perceptions and their influences on choices.

The methodology requires the estimation of an integrated multi-equation model consisting of a discrete choice model and the latent variable model's structural and measurement equations. The approach uses maximum likelihood techniques to estimate the integrated model, in which the likelihood function for the integrated model includes complex multi-dimensional integrals (one integral per latent construct). Estimation is performed either by numerical integration or simulation (MSM or SMLE), and requires customized programs and fast computers.

Three applications of the methodology are presented. The findings from the case studies are that implementation of the integrated choice and latent variable model framework results in: improvements in goodness of fit over choice models without latent variables, latent variables that are statistically significant in the choice model, and a more satisfying behavioral representation. Application of these methods require careful consideration of the behavioral framework, and then design of the data collection phase to generate good indicators and causal variables that support the framework.

To conclude, we note that the methodology presented here and the empirical case studies have merely brought to the surface the potential for the integrated modeling framework. Further work is needed to assess ramifications and to transcribe the methodological developments from an academic setting to practical applications, including investigation in the following areas:

Behavioral Framework: By integrating latent variable models and choice models, we can begin to reflect behavioral theory that has here-to-for primarily existed in descriptive flow-type models. The behavioral framework and the methodology we present needs to be extended to further bridge the gap between behavioral theory and statistical models. For example, including memory, awareness, process, feedback, temporal variables, tastes, goals, context, etc. in the framework.

Validation: The early signs indicate that the methodology is promising: the goodness of fit improves, the latent variables are significant, and the behavioral representation is more satisfying. For specific applications it would also be useful to conduct validation tests, including tests of forecasting ability, consequences of misspecifications (e.g., excluding latent variables that should be present), and performance comparisons with models of simpler formulations.

Identification: Other than the methods we present for identification (the *Three-step Rule*, the use of synthetic data, and the evaluation of the Hessian), there are no additional rules for identification of the general formulation of the integrated choice and latent variable models. Similar to the way that necessary and sufficient rules were developed for LISREL, the knowledge base of identification issues for the integrated model must be expanded.

Computation: Application of this methods is computationally intensive. Investigation of techniques such as parallel computing, particularly for estimation by simulation, would greatly ease the application of such models.

The approach presented in this paper is a flexible, powerful, and theoretically grounded methodology that will allow the modeling of complex behavioral processes. Now we need to further explore its potential.

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